HW2

Ch. 3 Problems 9, 13, 16, 20, 23 plus CYOP

3-9. You are given position and time in reference frame 1. Find the position and time using a) Lorentz transforms and b) using Galilean transforms in reference frame 2.

$\vec{r_1} = (1.0, -3.0, 0.00)km \quad t_1 = 4.0\mu s \qquad v_{21} = 0.98C$

So we have

$\beta = 0.98 \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}}$

$= 5.025$

$x_2 = \gamma(1km - 0.98c \times 4.0\,\mu s)$

$= -884.4\,m$

$y_2 = y_1 \qquad z_2 = z_1$

$t_2 = \gamma(t_1 - \beta/c\, x_1)$

$= 3.685\,\mu s$

Galilean -

$x_2 = x_1 - v t_1$

$= -176\,m$

the rest is same

3-13 At the origin with t=0 in both frames, a clock in frame S_2 ticks---we are given that γ=100. Captain Kirk, in Frame 2, hears the clock riding with him tick 1.00s later. What do we measure in frame 1?

So $x_1=x_2=0$, $t_1=t_2=0$, but $t_2'=1.00s$. With the clock still at $x_2'=0$

We need to use a reverse transformation to get t_1'

$$t_1' = \gamma\left(t_2' + \frac{\beta}{c}x_2'\right)$$

$$= 100(1.00\,s + 0) = 100\,s$$

Where does the clock tick x_1'

The speed of frame 2 with respect to 1 is very close to "c".

$$100 = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$v \approx c$ DO IT

SO I'LL USE C

$$x_1' = \gamma(x_2' + v\,t_2)$$

we already took into account

$$= 100(0 + 3\times10^8\,m/s\ 1\,s)$$

$$= 3.00\times10^{10}\,m$$

good approx here.

3-16 Show that the matrix form is a good representation of the Lorentz transforms.

Just multiply it out!

$$\begin{pmatrix} x_2 \\ y_2 \\ z_2 \\ ict_2 \end{pmatrix} = \begin{pmatrix} \gamma & 0 & 0 & ic\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -i\beta\gamma & 0 & 0 & \gamma \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \\ z_1 \\ ict_1 \end{pmatrix}$$

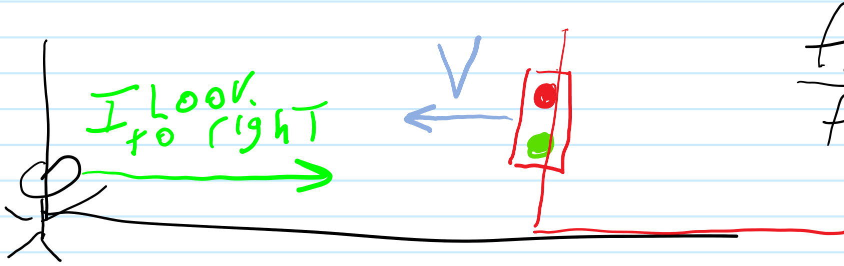
$$x_2 = \gamma x_1 + (ic\beta)(ict_1)$$
$$= \gamma(x_1 - vt_1)$$ ✓

$$y_2 = y_1 \qquad z_2 = z_1$$ ✓

$$ict_2 = -i\beta\gamma x_1 + \gamma(ict_1)$$
$$t_2 = \gamma\left(t_1 - \frac{\beta}{c} x_1\right)$$ ✓

3-20 You speed (in your starship) toward a galactic red light. The light you see Doppler shifts into the green---so you see a green light. Oops. How fast must you go?

f_{red}=4.5x10^{14} Hz and f_{green} =5.8x10^{14} Hz



$$\frac{f_{dopp}}{f_0} = \frac{1}{\gamma} \frac{1}{1 + \frac{v}{c}\cos\theta}$$

I am sitting still in my reference frame, asking how fast I should observe the traffic light moving toward me (the source is moving with respect to the observer). The angle between the direction I look (point into the light) and the direction the light moves is 180°

$$\frac{f_g}{f_R} = \frac{\sqrt{1-\beta^2}}{1-\beta}$$

$$1.289 = \frac{\sqrt{(1-\beta)(1+\beta)}}{1-\beta}$$

$$= \sqrt{\frac{1+\beta}{1-\beta}} \longrightarrow \text{square \& solve } \beta$$

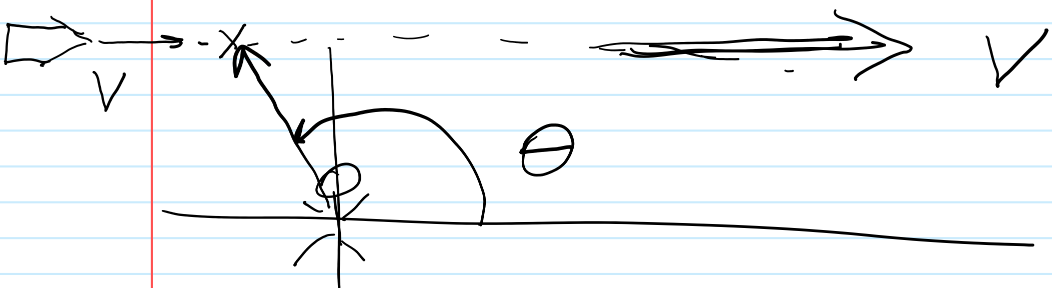
$$\beta = 0.2485$$

$$v = 7.456 \times 10^7 \text{ m/s}$$

Beat that Mav.
I feel the need for speed!

3-23

A spaceship moves at a speed of **0.200c.** Passing you by while you stand, as always at the origin. Let the spaceship move along a straight line path as shown. You want to find "position" (not really---angle is what you want) such that there is no Doppler shift. IT IS NOT 90, LIKE IT USED TO BE FOR GALILEAN!



So, the Doppler shifted frequency is required to be the same as the original frequency.

$$1 = \frac{\sqrt{1 - 0.2^2}}{1 + 0.2 \cos\theta}$$

$$1 + 0.2 \cos\theta = 0.979796$$

$$\cos\theta = -0.101021$$

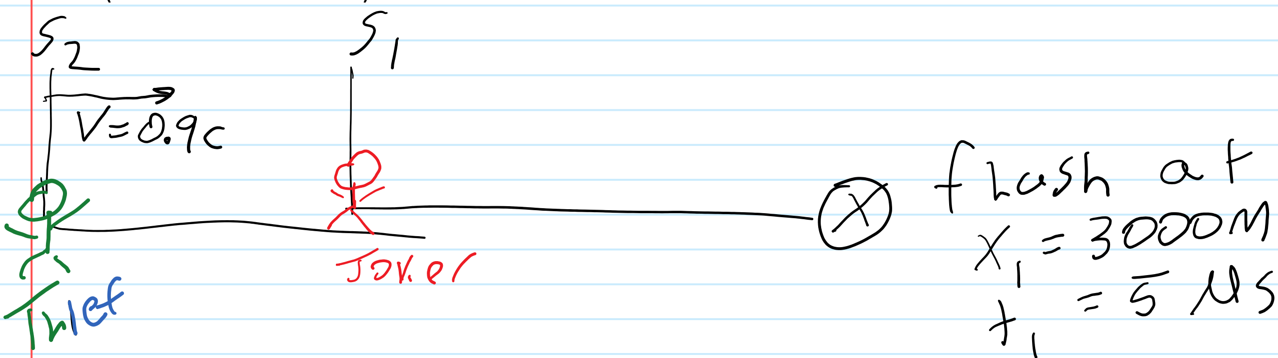
$$\theta = 95.798^\circ$$

This will differ depending on the speed. Think about ---how do you measure this, or measure the Doppler effect at any "point". Emission of several peaks (I must measure several peaks) occurs over a range of locations, hence the frequency is always changing. Transverse Doppler effect at 90---has been a critical test of the validity of special relativity.

Colbert CYOP--Follow the light

We will have an event occur, say a flash of light will be emitted from a watchtower at some location along the x axis (let's keep it on x to keep things simple). Two observers will be making measurements with what happens. Let's call one the "joker" in frame S1 with the watchtower not moving at all, and the other observer is in frame S2 moving in the usual way at a speed of 0.900c---I'll call this observer "thief".

a) Given x1=3000m and t1=5.000microseconds, we first want to find x2 and t2 (more to follow below)



$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = 2.294$$

$$x_2 = \gamma(3000 - 0.9c \times 5\times10^{-6})$$
$$= 3785m$$

$$t_2 = \gamma\left(t_1 - \frac{0.9}{c}\times 3000\right)$$
$$= -9.176\,\mu s$$

Is there some kinda confusion----The Thief says the event occurred before frame 2 crossed by frame 1 origin, but the Joker says this event happened much later. THERE IS NO INCONSISTENCY HERE. WE MUST FOLLOW THE LIGHT........WALK INTO THE LIGHT.

b) Let's ask "What is the transit time for light to leave from where the event occurs and travel to reach each observer"? Then we can discuss.

$$\Delta t_{1\,transit} = \frac{x_1}{c} = 10.0\,\mu s$$

$$\Delta t_{2\,transit} = \frac{x_2}{c} = 12.617\,\mu s$$

c) When and where are each observer when they received their own respective pulse of light (the light pulse that was emitted from the watchtower and then travelled outward like dipping finger in water)?

$$t_{1_{Received}} = 5\,\mu s + \text{transit}$$
$$= 15\,\mu s$$

$$t_{2_{Received}} = -9.176\,\mu s + 12.617\,\mu s$$
$$= 3.44\,\mu s$$

As for "where" they are both at the origin of their own frames.

d) I can ask--where does the Joker see the Thief (and vice versa) at the time the Joker receives the light (crosses the signal the one and only emanating ring of light).

Position of Frame 2 (thief) according To Joker at time 15 microsec $= 0.9c * t_{1_{Received}}$

$= 4050\ m$

Discussion: At this time, the Joker says the Thief is well ahead of him/her-and certainly received the light signal at some earlier time.

Position of Frame 1 (Joker) According to Thief at time Thief receives signal $= -0.9c * 3.44\,\mu s$

$-928.8\ m$

Discussion: OK--So the Thief agrees, the Joker is behind him even at the earlier time when the Thief passed through the ring of light. The Thief keeps moving ahead, and the Joker falls back by even more by the time the Joker passes the ring of light.

All I have done so far is argue that the order of events actually makes sense.   To finish this up, we can ask the following.

According to the Thief--how long does it take for light to now travel from the Thief to the Joker, and where is the Joker at that time? Then does this agree with transformation---it does.

We have followed the light.

I'll let you finish up--"The hours getting late".